CSIT 241 Spring 2002

Proof Techniques

• To prove that a statement of the form *if p, then q* (or $p \to q$) is false, it suffices to come up with an example (called *counterexample*) in which p is **true** and q is **false**.

• To prove that a statement of the form *if p, then q* (or $p \to q$) is true, you can do that either by a direct method or by an indirect method.

Direct Method: You assume p is **true** and then prove q is **true**.

Indirect Method: Here we'll present *proof by contradiction.* A special case of proof by contradiction is *proving the contrapositive* of *if p, then q* is **true**; i.e. proving *if not(q), then not(p)* is **true** (i.e. proving $\sim q \to \sim p$ is a true statement) . To prove the contrapositive, you assume q is **false** and then prove p is **false**. In general, to prove *if p, then q* by contradiction, you assume q is false, and then you reach something contradicting p or a known mathematical fact.

Mathematical Induction (Weak Form)

To prove that a formula is true for every integer greater than or equal to M, you have to do the following

1. Prove the formula is true for M.
2. Assume the formula is true for some integer k greater than M.
3. Depend on the assumption in the previous part (and may be you need also to depend on other known facts) to prove the formula is true for $k + 1$.

If you do all the above, then the principle of mathematical induction states your formula is true for every integer greater than or equal to M.