

CSIT 241 - Examples

1. A state wants to produce license plates consisting of 4 uppercase letters, a space, and three digits (zero is excluded). If repetition is allowed (i.e. you can repeat digits/letters), how many different license plates are possible?

Solution: There are $(26)^4$ ways to choose 4 uppercase letters and 9^3 ways to choose 3 nonzero digits. Thus, by the multiplication principle, there are $(26)^4 \cdot 9^3$ possible different license plates.

2. An instructor has divided his class into 8 groups. Each group has to give a presentation. The instructor wants the presentations to be given in the last 3 classes of the semester. He wants 3 presentations each day except the last day in which he wants only two presentations. In how many ways can this be done?

Solution: This is exactly as the question: In how many ways can 8 people wait in a line? Or in how many ways can we put 8 distinct balls in 8 distinct boxes at most one ball a box? So, the answer is $8!$ (or $P(8, 8)$).

3. A systems administrator decided to make a password consist of 7 characters, the first has to be from the set $\{A, B, C, D\}$ and the remaining 6 characters can be either lowercase English alphabets or digits. How many different passwords are possible?

Solution: We can first choose the first character. There are 4 possible ways to do that. Then choose the remaining 6 characters. There are $(26 + 10)^6$ possible ways to do that. Thus, by the multiplication principle, the answer is $4 \cdot (36)^6$.

4. In how many ways can you arrange the letters of the word BANANA (i.e. how many distinct permutations of the letters of BANANA are there)?

Solution: $\frac{6!}{3! \cdot 2!}$. We have to divide by $3!$ because A appears 3 times and by $2!$ because N appears twice.

5. A committee consists of 4 women and 6 men has to be chosen from a group of 20 women and 30 men. How many different committees can be chosen?

Solution: First choose the women. There are $C(20, 4)$ ways to do that. Then choose the men. There are $C(30, 6)$ ways to do that. Thus, by the multiplication principle, the answer is $C(20, 4) \cdot C(30, 6)$.

6. In how many ways can a person choose 4 CDs from the top ten list if repetition is allowed and if there are at least 4 CDs from each one of the top ten?

Solution: This is the same as asking: In how many ways can you put 4 identical balls into 10 distinct boxes any number in a box? The answer is: $C(10+4-1, 4)$.

7. In how many ways can the letters of the English alphabet be arranged so that there are exactly 10 letters between a and z?

Solution: There $P(24, 10)$ ways of choosing the 10 letters between a and z. Now look at those 10 letters and at a and z as one object. Then, you have now 14+1 objects to arrange. The number of different arrangements of them is $15!$. But, also either a will be first or z will be first, so you have to multiply by 2. Thus, by the multiplication principle, the answer is: $2 \cdot P(24, 10) \cdot 15!$.

8. A man, a woman, a boy, a girl, a dog, and a cat, are walking down a road one after another. In how many ways can this happen if the dog has to be between the man and the boy?

Solution: Think of the man, the boy, and the dog, as one object. Thus, you have 4 objects to order. You can do that in $4!$ different ways. Now since either the boy will be first or the man will be first, then you'll have to multiply by 2. Thus, the answer is: $2 \cdot 4!$.

9. In how many ways can 8 books be split among Jay, Mary, and Chris, if Jay has to get 4 books, and Mary and Chris each gets two books?

Solution: The question is the same as: In how many ways can you arrange the letters of the word JJJJMMCC? So, the answer is $\frac{8!}{4! \cdot 2! \cdot 2!}$.

10. Look at all the examples we did in class. It is very important to do that.