Matrices (Chapter 3)

Definition: A *matrix* is a two-dimensional array of numbers.

- If a matrix A has m rows and n columns, then we say A is m by n or $m \times n$.

Example: Let

$$A = \begin{bmatrix} 3 & 5 & -1 \\ 7 & 3 & 9 \end{bmatrix}.$$

Then A is 2×3.

Let A be an $m \times n$ matrix.

1. If $m = n$, then A is called a *square* matrix.
2. If $m = 1$ or $n = 1$, then A is called a *vector*.
3. $A(i, j)$ (or a_{ij}) is the element of A in row i and column j.
4. $A(i, *)$ is the i^{th} row of A.
5. $A(*, j)$ is the j^{th} column of A.
6. If all elements of A are equal to zero, then A is called the zero matrix and it is denoted by Z or 0. The zero matrix is the additive identity for matrices.

Example: Let

$$A = \begin{bmatrix} 3 & 5 & -1 \\ 7 & 3 & 9 \end{bmatrix}.$$

Then

- $A(1, 2) = 5$, $A(2, 1) = 7$ ($a_{12} = 5$, $a_{21} = 7$).
- $A(1, *) = \begin{bmatrix} 3 & 5 & -1 \end{bmatrix}$.
- $A(*, 1) = \begin{bmatrix} 3 \\ 7 \end{bmatrix}$.

Addition of Matrices and Scalar Multiplication

Let A and B be $m \times n$ matrices and let r be a scalar. Then

1. $(A+B)(i,j) = A(i,j) + B(i,j)$.
2. $(rA)(i,j) = rA(i,j)$.

Example: Let

$$A = \begin{bmatrix} 3 & 4 \\ 5 & 6 \\ 7 & 8 \end{bmatrix} \text{ and } B = \begin{bmatrix} -1 & 2 \\ 0 & 3 \\ 4 & 5 \end{bmatrix}.$$

Then

$$A + B = \begin{bmatrix} 2 & 6 \\ 5 & 9 \\ 11 & 13 \end{bmatrix} \text{ and } -2A = \begin{bmatrix} -6 & -8 \\ -10 & -12 \\ -14 & -16 \end{bmatrix}$$

Facts: Let A and B be matrices of the same size and let r be a scalar. Then

1. $A + B = B + A$.
2. $A + (B + C) = (A + B) + C$.
3. $r(A + B) = rA + rB$.

Special Matrices

Let A be an $n \times n$ matrix. Then

1. The main diagonal of A is the elements: $a_{11},\ a_{22}, \cdots,\ a_{nn}$.
2. A is called *diagonal* if every off main diagonal element of A is zero.
3. A is called the *identity* matrix (usually denoted I) if it is diagonal and the elements of the main diagonal are all equal to 1.
4. A is called *uppertriangular* if every element of A below the main diagonal is equal to zero.
5. A is called *lowertriangular* if every element of A above the main diagonal is equal to zero.