Review

Review the following: definition of functions, domain, target, range, image, preimage, one-to-one (injective), onto (surjective), bijection (one-to-one correspondence), composition of functions.

Review from Calculus:

Prove or disprove the following: Let $f: A \longrightarrow B$ and $g: B \longrightarrow C$ be functions, let $\phi \neq S \subseteq A$ and $\phi \neq T \subseteq B$, let G be preimage of the image of S and H the image of the preimage of T, and let a, b, c, and d be real numbers and let $a < b$ and $c < d$. (For the first three parts, if the statement is false and if one direction is true, indicate that, and give a counterexample to show the other direction is false.) Then

(1) $g \circ f$ is one-to-one iff g and f are one-to-one.
(2) $g \circ f$ is onto iff g and f are onto.
(3) $g \circ f$ is bijection iff g and f are bijection.
(4) $G = S$. If false, indicate if one of these two sets is a subset of the other.
(5) $H = T$. If false, indicate if one of these two sets is a subset of the other.
(6) Prove that the function $f: (\mathbb{N} \cup \{0\}) \times (\mathbb{N} \cup \{0\}) \longrightarrow \mathbb{N} \cup \{0\}$, defined by $f(k, n) = 2^k(2n + 1) - 1$ is one-to-one.
(7) Give an example of a bijection from (a, b) to (c, d).
(8) Give an example of a bijection from $\mathbb{R}$ to (a, ∞).
(9) Give an example of a bijection from $(0, 1)$ to (a, ∞).