

Answers for Assignment 1

(1) Let

$$A = \begin{bmatrix} 2 & 3 & 4 & 5 & 7 \\ 3 & 4 & 5 & 6 & 3 \\ 4 & 5 & 6 & 2 & 4 \\ 5 & 6 & 2 & 3 & 1 \\ 6 & 2 & 3 & 4 & 2 \end{bmatrix}.$$

Write A as a sum of a symmetric matrix and a skew-symmetric matrix. I.e. find a symmetric matrix B and a skew-symmetric matrix C such that $A = B + C$.

Answer: Recall that we proved in class that every matrix can be written as a sum of a symmetric matrix and a skew-symmetric matrix and we explained how to get the symmetric part and the skew-symmetric part. Using that, we get $B = \frac{1}{2}(A + A^T)$ and $C = \frac{1}{2}(A - A^T)$. Thus,

$$B = \begin{bmatrix} 2 & 3 & 4 & 5 & \frac{13}{2} \\ 3 & 4 & 5 & 6 & \frac{5}{2} \\ 4 & 5 & 6 & 2 & \frac{7}{2} \\ 5 & 6 & 2 & 3 & \frac{5}{2} \\ \frac{13}{2} & \frac{5}{2} & \frac{7}{2} & \frac{5}{2} & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{1}{2} \\ 0 & 0 & 0 & 0 & \frac{-3}{2} \\ \frac{-1}{2} & \frac{-1}{2} & \frac{-1}{2} & \frac{3}{2} & 0 \end{bmatrix}.$$

(2) Prove that every skew-symmetric matrix of odd order is singular.

Answer: Done in class.

(3) Let

$$A = \begin{bmatrix} 2 & 3 & 2 \\ 2 & 2 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

Find A^{-1} using the adjoint method.

Answer:

$$A^{-1} = \begin{bmatrix} \frac{-1}{3} & \frac{1}{3} & \frac{1}{3} \\ \frac{-1}{3} & \frac{4}{3} & \frac{-2}{3} \\ \frac{4}{3} & \frac{-7}{3} & \frac{2}{3} \end{bmatrix}$$

(4) Solve the following system by

(a) The inverse method (i.e. by using the inverse of the coefficient matrix).

Answer: Write the system in the form $Ax = b$. Now A is the same as matrix A in the previous question. Hence, A is invertible and its inverse is found in the previous question. Therefore, the solution x is

$$x = A^{-1}b = \begin{bmatrix} \frac{5}{3} \\ \frac{2}{3} \\ \frac{-8}{3} \end{bmatrix}.$$

(b) Gramer's rule. Answer: $x_1 = \det(A_1)/\det(A)$, $x_2 = \det(A_2)/\det(A)$, $x_3 = \det(A_3)/\det(A)$, where A is the matrix of coefficients (same as in (3) and 4(a)), and

$$A_1 = \begin{bmatrix} 0 & 3 & 2 \\ 2 & 2 & 1 \\ 3 & 1 & 1 \end{bmatrix}, \quad A_2 = \begin{bmatrix} 2 & 0 & 2 \\ 2 & 2 & 1 \\ 3 & 3 & 1 \end{bmatrix}, \quad A_3 = \begin{bmatrix} 2 & 3 & 0 \\ 2 & 2 & 2 \\ 3 & 1 & 3 \end{bmatrix}.$$

$$\text{Thus, } x_1 = \frac{-5}{-3} = \frac{5}{3}, \quad x_2 = \frac{-2}{-3} = \frac{2}{3}, \quad \text{and } x_3 = \frac{8}{-3} = \frac{-8}{3}.$$

$$2x_1 + 3x_2 + 2x_3 = 0.$$

$$2x_1 + 2x_2 + x_3 = 2.$$

$$3x_1 + x_2 + x_3 = 3.$$

(5) Find the determinant of the following matrix using cofactors:

$$A = \begin{bmatrix} 5 & 0 & 0 & 0 \\ 9 & 2 & 3 & 2 \\ 8 & 2 & 2 & 1 \\ 7 & 3 & 1 & 1 \end{bmatrix}$$

Answer: -15.

(6) Let x be an $n \times 1$ vector such that $x^T x = 2$. Find $(I_n - xx^T)^{-1}$.

Answer: Done in class.