Questions abour Recurrence Relations

For the following questions, use only the methods/theorems I've covered in class:

1. $a_n = 5a_{n-1} - 6a_{n-2}$, $a_0 = 6$, $a_1 = 8$, $n \geq 2$.
2. $a_n = 5a_{n-2}$, $a_0 = 1$, $a_1 = 0$, $n \geq 2$.
3. $2a_n + a_{n-1} = 6a_{n-2}$, $a_0 = 1$, $a_1 = 0$, $n \geq 2$.
4. $a_n - 5a_{n-2} = -6a_{n-4}$, $n \geq 4$. (*Here just find the general solution.*)
5. $a_n + 2a_{n-1} = -a_{n-2}$, $a_0 = 1$, $a_1 = 0$, $n \geq 2$.
6. $a_n - 2a_{n-1} = -a_{n-2}$, $a_0 = 1$, $a_1 = 0$, $n \geq 2$.
7. $a_n = 4a_{n-2} + 3n$, $a_0 = 1$, $a_1 = 0$, $n \geq 2$.
8. $a_n = 5a_{n-1} - 6a_{n-2} + 2.5^n$, $a_0 = 6$, $a_1 = 8$, $n \geq 2$.
9. $a_n - 6a_{n-1} = 8a_{n-3} - 12a_{n-2}$, $n \geq 3$. (*Here just find the general solution.*)
10. $a_n - a_{n-1} = -12a_{n-3} + 8a_{n-2}$, $n \geq 3$. (*Here just find the general solution.*)
11. $a_{n-1}^2 = a_n a_{n-2}$, $a_0 = 1$, $a_1 = 2$, $n \geq 2$.
12. $a_n = \frac{a_{n+2} - 5a_{n+1}}{-6}$, $a_0 = 6$, $a_1 = 8$, $n \geq 0$.
13. Here find both a_n and b_n.
$$a_n = a_{n-1} + b_{n-1},\ a_{n-1} = b_n + b_{n-1},\ n \geq 1,\ a_0 = 1,\ b_0 = 2.$$
14. $a_n = 5na_{n-1}$, $a_0 = 1$, $n \geq 1$.
15. $a_n = 5^n a_{n-1}$, $a_0 = 1$, , $n \geq 1$.
16. $\sqrt{a_n} = \sqrt{a_{n-1}} + 2\sqrt{a_{n-2}}$, $a_0 = a_1 = 1$, $n \geq 2$.
17. $a_n = \sqrt{\frac{a_{n-2}}{a_{n-1}}}$, $a_0 = 8$, $a_1 = 2^{\frac{-3}{2}}$, $n \geq 2$.
18. $a_n^2 = 5a_n a_{n-1} - 6a_{n-1}^2$, $a_0 = 1$, $a_1 = 3$.
19. $a_n = 5\sqrt{a_n a_{n-1}} - 6a_{n-1}$, $a_0 = 1$, $a_1 = 9$.

Answers

1) $a_n = 5.2^{n+1} - 4.3^n$, $n \geq 0$.

2) $a_n = \frac{1}{2}[\sqrt{5}^n + \sqrt{-5}^n]$, $n \geq 0$.

3) $a_n = \frac{4}{7}(\frac{3}{2})^n + \frac{3}{7}(-2)^n$, $n \geq 0$.

4) $a_n = \beta_1\sqrt{3}^n + \beta_2 - \sqrt{3}^n + \beta_3\sqrt{2}^n + \beta_4 - \sqrt{2}^n$, $n \geq 0$.

5) $a_n = (-1)^n(1-n)$, $n \geq 0$.

6) $a_n = 1 - n$, $n \geq 0$.

7) $a_n = \frac{11}{4}2^n + \frac{11}{12}(-2)^n - n - \frac{8}{3}$, $n \geq 0$.

8) $a_n = \frac{80}{3}2^n - 29.3^n + \frac{1}{3}5^{n+2}$, $n \geq 0$.

9) $a_n = \beta_1 2^n + \beta_2.n.2^n + \beta_3.n^2.2^n$, $n \geq 0$.

10) $a_n = \beta_1 2^n + \beta_2.n.2^n + \beta_3.(-3)^n$, $n \geq 0$.

11) $a_n = 2^n$, $n \geq 0$.

12) Same as (1).

13) As we did in class, you can get a recurrence relation for $\{a_n\}$, which is

$$a_n = 2a_{n-2},\ n \geq 2,\ a_0 = 1,\ a_1 = 3.$$

Now solve this for a_n, then ssubstitute the result in the second original equation and solve for b_n.

14) $a_n = 5^n.n!$, $n \geq 0$.

15) $a_n = 5^{\frac{n(n+1)}{2}}$, $n \geq 0$.

16) $a_n = \frac{1}{9}[2^{n+1} + (-1)^n]^2$, $n \geq 0$.

17) $a_n = 2^{2^{-n}+2.(-1)^n}$, $n \geq 0$.