CSCE 235 **Feb 2, 01**

Quiz #2

(0 points) Name: ..... (0 points) SSN: .....

Do **All** of the following questions and show your work.

Question 1: (6 points) Let $A = \mathbb{Z} \times \mathbb{Z}$, and let $B = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1\}$. Find $A \cap B$.

Question 2: (7 points) Let $\{u_n\}_{n=1}^{\infty}$ be defined by

$$u_n = \tfrac{2}{3} - (\tfrac{1}{6})(\tfrac{1}{4^{n-2}}), \forall n \in \mathbb{N}.$$

Let $A = (-5, \frac{11}{3}] \cap [\frac{7}{4}, 5) \cap \mathbb{Z}^+$ and let $S = A - \{-4, 7\}$. Find

$$\Sigma_{i \in S}\ u_i$$

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Question 3: (7 points) Let

$$A = \{(-1, 2), (4, 5), (0, 0), (6, -5), (5, 1), (4, 3)\}$$

$$B = \{b \mid b = k^2 \text{ for some } k \in \mathbb{Z} \text{ and } (a, b) \in A \text{ for some } a\}.$$

$$C = \{x - 4 \mid x \in Z \text{ and } \tfrac{x^2-5x+6}{-1576} \geq 0\}.$$

Find $(C \cup B) \cap \{\{-3, 1, 2\}, \{1\}, 0, 5, 4, \{2\}, \{0\}, -2, -1, \phi\}$.

Question 4: (6 points) Prove by mathematical induction:

$$(1 + 2 + 3 + ... + n)^2 = 1 + 2^3 + ... + n^3, \forall n \in \mathbb{N}.$$