

Quiz #2

(0 points) Name: (0 points) SSN:

Do **All** of the following questions and show your work.

Question 1: (3 points) Let A be a subset of $\mathbb{N} \times \mathbb{N}$ with the properties:

(a) $(1, 1) \in A$ and

(b) $(a, b) \in A \longrightarrow (a + 1, b)$ and $(a + 1, b + 1)$ are both in A .

Let $B = \{(m, n) \in \mathbb{N} \times \mathbb{N} \mid m \geq n\}$.

Is $B \subseteq A$?

Question 2: (5 points) Let $\{u_n\}_{n=1}^{\infty}$ be defined by

$$u_n = \frac{2}{3} - \left(\frac{1}{6}\right)\left(\frac{1}{4^{n-2}}\right), \forall n \in \mathbb{N}.$$

Let $A = (-5, \frac{11}{3}] \cap [\frac{7}{4}, 5) \cap \mathbb{Z}^+$ and let $S = A - \{-4, 7\}$. Find

$$\sum_{i \in S} u_i$$

Question 3: (6 points) Prove by mathematical induction:

$$(1 + 2 + 3 + \dots + n)^2 = 1 + 2^3 + \dots + n^3, \forall n \in \mathbb{N}.$$

Question 4: (6 points) Let

$$A = \{(-1, 2), (4, 5), (0, 0), (6, -5), (5, 1), (4, 3)\}$$

$$B = \{b \mid b = k^2 \text{ for some } k \in \mathbb{Z} \text{ and } (a, b) \in A \text{ for some } a\}.$$

$$C = \{x - 4 \mid x \in \mathbb{Z} \text{ and } \frac{-157}{7x^2 - 35x + 42} \geq 0\}.$$

Find $(C \cup B) \cap \{-3, 1, 2\}, \{1\}, 0, 5, 4, \{2\}, \{0\}, -2, -1, \phi\}$.