

Due: Monday, April 2, 01

Question 1: Do **ONLY** 4 of the following parts.

In how many ways can you order 7 math books, 9 computer science books, and 6 physics books on a shelf if

- (a) all math, computer science, and physics books are distinct?
- (b) all math, computer science, and physics books are distinct, and the math books are to be together?
- (c) all math books are identical and all computer science and physics books are distinct?
- (d) only 4 of the math books are identical and all computer science and physics books are distinct?
- (e) Repeat part (b) for a circular shelf.

Question 2: Do **ONLY** 5 of the following parts.

- (a) In how many ways can you distribute 17 distinct balls into 25 distinct boxes at most one ball to a box?
- (b) In how many ways can you distribute 17 distinct balls into 25 distinct boxes? (Here there is no limit on the number of balls in each box.)
- (c) In how many ways can you distribute 17 red (identical) balls into 25 distinct boxes at most one ball to a box?
- (d) In how many ways can you distribute 17 red (identical) balls into 25 distinct boxes? (Here there is no limit on the number of balls in each box.)

- (e) In how many ways can you distribute 10 red (identical) balls and 7 blue (identical) balls into 25 distinct boxes at most one ball to a box?
- (f) In how many ways can you distribute 10 distinct red balls and 7 blue (identical) balls into 25 distinct boxes at most one ball to a box?
- (g) In how many ways can you distribute 10 red (identical) balls and 7 blue (identical) balls into 25 distinct boxes? (Here there is no limit on the number of balls in each box.)
- (h) In how many ways can you distribute 10 distinct red balls and 7 blue (identical) balls into 25 distinct boxes? (Here there is no limit on the number of balls in each box.)
- (i) A bag contains 10 red distinct balls and 7 blue distinct balls. In how many ways, can you draw 5 red and 4 blue?

Question 3:

- (a) How many permutations of the letters *PRODUCTIVE* contain a permutation of *PRO*?
- (b) What is the number of permutations of the word *PRODUCT* if no letter is to keep its original position?

Question 4:

- (a) Find the middle term in the binomial expansion of $(3y - 2x)^{34}$. What is the coefficient of $x^{11}y^{23}$ in the previous expansion?
- (b) Find the coefficient of x^{20} and the coefficient of x^{21} in the binomial expansion of $(4x^2 - \frac{1}{x})^{40}$.

Question 5: Do **ONLY** 2 of the following parts.

(a) 100 points are to be distributed on 9 questions. If each question has to be out of at least 4. In how many ways can you distribute the points?

(b) What is the number of integer solutions of

$$x + y + z = 73, x \geq 0, y \geq 3, z > 7?$$

(c) A 6-member committee has to be chosen from a group of 9 mathematicians and 13 computer scientists. How many committees can be formed if the committee has to include two mathematicians only? And how many committees can be formed if the committee has to include at least two mathematicians?

Question 6: Do **ONLY** 5 of the following parts.

(a) Find the number of odd 6-digit octal numbers if repetitions are not allowed?

(b) Find the number of 6-digit octal numbers that read the same from either side.

(c) Find the number of 7-digit octal strings that read the same from either side.

(d) If repetitions are not allowed, what is the number of 6-digit octal numbers that contain a 2 and a 5?

(e) If repetitions are not allowed, what is the number of 6-digit octal numbers that contain (*as a substring*) either 25 and 52?

(f) If repetitions are not allowed, what is the number of 6-digit octal numbers that start (from the left) with a 5 and end (from the right) with a 2 and which do not contain a 4?

(g) How many 6-digit octal numbers contain a 2 and do not contain a 5?

(h) If repetitions are not allowed, what is the number of 6-digit octal numbers that do not contain (*as a substring*) neither 25 nor 52?