CSCE 235 **Spring 01**

Practise Questions

Question 1:

(a) Prove that the function $f : (\mathbb{N} \cup \{0\}) \times (\mathbb{N} \cup \{0\}) \longrightarrow \mathbb{N} \cup \{0\}$, defined by $f(k, n) = 2^k(2n + 1) - 1$ is one-to-one.

(b) Prove that the function $f : (\mathbb{N} \cup \{0\}) \times (\mathbb{N} \cup \{0\}) \longrightarrow \mathbb{N} \cup \{0\}$, defined by $f(k, n) = 2^k(2n + 1) - 1$ is onto. (*You have to be very careful here.*)

(c) Find f^{-1}, where f is the function defined above.

(d) Let $g : (\mathbb{N} \times \mathbb{N}) \longrightarrow \mathbb{N}$, be defined by $g(k, n) = 2^k(2n + 1) - 1$. Is g one-to-one? Is it onto?

Question 2: Determine whether the function $f : \mathbb{Z} \times \mathbb{Z} \Longrightarrow \mathbb{Z}$, defined by

$$f(n, m) = 89\ n + 246\ m$$

is one-to-one and onto.

Question 3: Let U be the unit disc in $\mathbb{R}^2$, $X = \{x + \frac{1}{5} \mid x \in Z\}$ and let $Y = \mathbb{Z} \cup \{\frac{1}{5}\}$. Also, let $G = \{(x, y) \in X \times X | \ |x| \leq \frac{4}{7}\}$, $A = U \cap (Y \times Y)$, $B = U \cap G$, and $C = U \cap (\mathbb{Z} \times \mathbb{Z})$. Find

(a) $A \triangle B$.

(b) $A \triangle C$.

(c) $B \triangle C$.

(d) $A \cup B$.

(e) $A \cup C$.

(f) $B \cup C$.

Question 4: Find $\mathcal{P}(\mathcal{P}(\{\phi, \{0, \{1\}, a\}\}))$.

Question 5: Let A be any set and let R be a relation on R.

(a) Find sufficient and necessary conditions for R to be symmetric and antisymmetric.

(b) Find sufficient and necessary conditions for R to be symmetric and not antisymmetric.

(c) Find sufficient and necessary conditions for R to be not symmetric and antisymmetric.

(d) Find sufficient and necessary conditions for R to be not symmetric and not antisymmetric.

(e) Find sufficient and necessary conditions for R to be equivalence and antisymmetric.

(f) Find sufficient and necessary conditions for R to be equivalence and not antisymmetric.

(g) If R is an equivalence relation, find a necessary and sufficient condition for the equivalence classes of R to consist of one element (each).