

**Question 1:** Let  $A = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 1\}$ ,  $B = \{(x, y) \in \mathbb{R}^2 \mid |x| \leq 1 \text{ and } |y| \leq 1\}$ . Let  $M = \mathbb{Z} \cup \{\frac{1}{2}\}$  and  $L = \{x \mid x = \frac{1}{2} + q, q \in \mathbb{Z}\}$ . Finally, let  $C = A \cap (M \times M)$ ,  $D = B \cap (M \times M)$ ,  $E = A \cap (L \times L)$ ,  $F = B \cap (L \times L)$ . Find

- (a)  $D - C$ .
- (b)  $F \cap D$ .

**Question 2:**

- (a) Give an example of a bijective function from  $\mathbb{Z}$  onto  $\mathbb{N}$ .
- (b) Show using the definition of one-to-one that the function  $f : \mathbb{N} \rightarrow [e^{\frac{1}{4}}, e)$  defined by  $f(x) = e^{\frac{x}{x+3}}$  is one-to-one.

**Question 3:** The following questions are unrelated.

- (a) Let  $R$  be the relation on  $\mathbb{Z}$  defined by

$$a R b \text{ if and only if } a = |b|^3.$$

Is  $R$  antisymmetric? Is it transitive? If the answer to any of them is yes, prove that. If the answer is not, then write down a counter example. Also, find  $R^{-1}$ .

- (b) Let  $R$  be the relation on  $\mathbb{Z}$  defined by

$$a R b \text{ if and only if } 3 - b \leq a - 2b < 1 + b.$$

Give a counter example to show that  $R$  is not transitive. Is  $R$  symmetric? If yes, prove it. If not write down a counterexample. Think (but do not hand in) about how to find  $R^{-1}$ .

- (c) Let  $R$  be the relation on  $\mathbb{Z}$  defined by

$$a R b \text{ if and only if } a - b \text{ is divisible by } 5.$$

This relation is an equivalence relation. Find [17].

**Question 4:** The following parts are unrelated.

- (a) Express the decimal number 578 in octal.
- (b) Express the hexadecimal number 1E2B in decimal.
- (c) Add the two binary numbers 10111 and 110011 using binary addition.

**Question 5:** Let the sequence  $a$  be defined as follows:

$$a_n = 1 + (-1)^n \text{ if } n \text{ is an odd natural number and } a_n = (n - 1) + n \text{ if } n \text{ is an even natural number.}$$

$$\text{Let } S_n = \sum_{k=1}^n a_k.$$

- (a) Find a general formula for  $S_n$ ,  $n \in \mathbb{N}$ .
- (b) Is  $\{a_n\}_{n=1}^{\infty}$  increasing? Is it decreasing? Explain.