## Graphs Practise Questions III

**In the following questions, all graphs are undirected and simple.**

**Question 1:**

Let $G = (V, E)$ be defined by $V = \{v_1, v_2, v_3, v_4, v_5\}$, and

$$E = \{(v_i, v_j) \mid \text{mod}(i, j) = 1\}.$$

Find $|E|$, the adjacency matrix of $G$, and $\overline{G}$. Also, determine whether $G$ has an Euler cycle or not.

**Question 2:** Let $G = (V, E)$, where $V = \{1, 2, 3, 4, 5\}$. Let the adjacency matrix of $G$ be $A$ and the adjacency matrix of $\overline{G}$ be $\overline{A}$, where

$$\overline{A} = \begin{bmatrix} 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

Now let $H = (V', E')$, where $V' = \{1, 2, 5\}$. Let the adjacency matrix of $H$ be $A'$ and the adjacency matrix of $\overline{H}$ be $\overline{A'}$, where

$$\overline{A'} = \begin{bmatrix} 1 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 1 \end{bmatrix}$$

Determine

(a) without graphing

(b) by graphing

whether $H$ is a subgraph of $G$ or not.

**Question 3:** Let $G = (V, E)$ be defined by $V = \{v_1, v_2, v_3, ..., v_{100}\}$, and

$$E = \{(v_2, v_1), (v_1, v_5), (v_2, v_3), (v_4, v_3), (v_4, v_5), ...\}.$$

Is $G$ bipartite? Explain.

**Question 4:** Let $A$ be the adjacency matrix of $G = (V, E)$ and let $|V| = 15$. Can the main diagonal of $A^2$ contain 14 and 0? Explain.

**Question 5:** Let $A$ be the adjacency matrix of $G = (V, E)$ and let the main diagonal of $A^2$ contain 8, 2, 0, 6, 2, 4, .... Does $G$ have an Euler cycle? Does it have a Hamiltonian cycle? Explain.

**Question 6:** Let $A$ be the adjacency matrix of $G = (V, E)$

(a) How many edges does $G$ have if the sum of the elements of $A$ is 40?

(b) How many edges does $G$ have if the sum of the elements of the main diagonal of $A^2$ is 40?

(c) What is the relationship between the sum of the elements of $A$ and the sum of the elements of the main diagonal of $A^2$.

**Question 5:** If the number of edges of an undirected simple graph $G$ is 50 and the number of edges of $\overline{G}$ is 140. How many vertices does $G$ have?