Graphs Practise Questions II

In the following, assume all graphs are simple and undirected.

Question 1: Prove that the only cyclic graph isomorphic to its complement is C_5.

Question 2: Let $G = (V, E)$ be undirected graph, prove that if G is isomorphic to its complement, then either $|V|$ or $|V| - 1$ is a multiple of 4.

Question 3: Prove that if the bipartite $G_1 = (V_1, E_1)$ is isomorphic to $G_2 = (V_2, E_2)$, then G_2 is also bipartite.

Question 4: Let S be a clique in $G_1 = (V_1, E_1)$, and assume that G_1 is isomorphic to $G_2 = (V_2, E_2)$ with an isomorphism f. Is $f(S)$ a clique in G_2?

Question 5: Prove that if $G = (V, E)$ is a complete bipartite grapg that is isomorphic to $H = (V', E')$, then H is also a complete bipartite graph.

Question 6: Give **four** different proofs for the identity:

$$1 + 2 + 3 + ... + n = \frac{n(n+1)}{2}, \forall n \in \mathbb{N}.$$

One of your proofs should be based on the number of edges of K_m (the complete graph on m vertices).

Question 7: All examples we did in class.