

Practise Questions for sections 1.3 and 1.4

Question 1: Let A be any set and let R be a relation on R .

- (a) Find sufficient and necessary conditions for R to be symmetric and antisymmetric.
- (b) Find sufficient and necessary conditions for R to be symmetric and not antisymmetric.
- (c) Find sufficient and necessary conditions for R to be not symmetric and antisymmetric.
- (d) Find sufficient and necessary conditions for R to be not symmetric and not antisymmetric.
- (e) Find sufficient and necessary conditions for R to be equivalence and antisymmetric.
- (f) Find sufficient and necessary conditions for R to be equivalence and not antisymmetric.
- (g) If R is an equivalence relation, find a necessary and sufficient condition for the equivalence classes of R to consist of one element (each).

Question 2: Let A be the set of all first degree polynomials with integer coefficients. Define the relation R on A to be

$$\{(f, g) \mid f \text{ and } g \text{ have the same slope}\}.$$

Is R an equivalence relation? If yes, then what are the equivalence classes of $f(x) = -999$ and of $g(x) = -999x + 777$.

Question 3: Let A be the set of all continuous functions. Define the relation R on A to be

$$\{(f, g) \mid f(0) = g(0) + 3k, k \in \mathbb{Z}\}.$$

Is R an equivalence relation on A ? If yes, then what are the equivalence classes of $f(x) = -999$ and of $g(x) = -999x^{777}$.

Question 4: Define the following relation R on \mathbb{R}^2 :

$$(a, b) \sim (c, d) \text{ iff } |3a - 2b| = |3c - 2d|.$$

Is R an equivalence relation on \mathbb{R}^2 ? If yes, then give five elements in the equivalence class of $(1, 2)$. Does $(\frac{1}{3}, \frac{5}{9}) \in \overline{(1, 2)}$? Is $\overline{(1, 2)} = \overline{(-1, 2)}$? Explain.

Question 5: Consider the poset (A, \subseteq) , where $A = \{\{1\}, \{1, 2\}, \{1, 3\}, \{1, 2, 4\}\}$.

- Find all maximum, minimum, maximal, and minimal elements of A .
- Is (A, \subseteq) a totally ordered set? Explain
- Draw the Hasse Diagram of A .
- Find the *lub* and the *glb* of $\{1, 2\}$ and $\{1, 3\}$.
- Is (A, \subseteq) a lattice? Explain.

Question 6:

- Do 3(f) and 4(f) page 61 (text).
- Do 5(b) page 61 (text).
- Do 18(b) page 56 (text).