Questions about Permutations, Combinations, and Derangements

(1) Evaluate $\sum_{k=0}^{n}(-1)^k C(n,k)3^{n-k}$.

(2) Use the Binomial Theorem to simplify 1.01^3.

(3) Prove each of the following with no more than one line

- $\sum_{k=0}^{n}\frac{1}{k+1}2^{k+1}C(n,k)=\frac{3^{n+1}}{n+1}$.
  Solution: Consider $(x+1)^n=\sum_{k=0}^{n}C(n,k)x^k$. Now take the integral of this equation with respect to x and then substitute in the result $x=2$.
- $\sum_{k=1}^{n}kC(n,k)=n2^{n-1}$. You have to use here a method different than that we used in class. Remember this was solved in class but by a different method.
  Solution: Consider $(x+1)^n=\sum_{k=0}^{n}C(n,k)x^k$. Now take the derivative of this equation with respect to x and then substitute in the result $x=1$.

(4) With no more than one line, evaluate $\sum_{k=1}^{n}k^2C(n,k)$.
Solution: Consider $(x+1)^n=\sum_{k=0}^{n}C(n,k)x^k$. Now take the derivative of this equation with respect to x twice and then substitute in the result $x=1$.

(5) You are given 33 points in the cartesian plane. No 3 of them are collinear. How many triangles can you make of these points if the vertices of each triangle have to be from the given points.
Solution: $C(33,3)$.

(6) 23 persons are to sit around a circular table. This qroup of people includes A and B who don't like each other and don't want to sit next to each other. In how many ways can the 23 persons sit.
Solution: (20)(21!). (22!-(2!. 21!))

(7) Prove that $D_n=(-1)^n+nD_{n-1}$.

(8) Use the above (what you've just proved) to prove in no more than two lines that D_n is even if n is an odd positive integer and odd if n is an even positive integer.