

Name :

SSN :

*Instructions:***1. Show your work and explain your answers.****2. Do only 6 of the first 8 questions AND****3. Do also the last two questions.****Question 1:** (10 points)

Thirty buses are to be used to transport 2000 people. Each bus has 80 seats. Assume one seat per passenger. Prove that one of the buses will have at least 14 empty seats.

Question 2: (10 points) Find the coefficient of x^{12} in the binomial expansion of $(1+2x^2)^{18}$.**Question 3:** (10 points) In how many ways can 12 red balls and 15 blue balls be placed in 37 different boxes with:

(a) At most one ball to a box?

(b) No limit on the number of balls in each box?

Question 4: (10 points) 1. In how many ways can 13 math books and 9 different physics books be arranged on a shelf if the physics books are to be together and if 5 of the math books are identical and the rest of the math books are non-identical?

Question 5: (10 points)

A 17-member committee has to be chosen from a group of 15 mathematicians, 12 computer scientists and 9 engineers. The committee has to include exactly 5 computer scientists and exactly 4 engineers. In how many ways such a committee can be formed?

Question 6: (10 points) In how many ways can you rearrange the letters of the word Fortran?

Question 7: (10 points)

(a) How many 8-digit numbers have no repeated digits?

(b) How many odd 8-digit numbers have no repeated digits?

Question 8: (10 points) In a class of 50 people, 25 know Java, 17 know Visual Basic, and 8 know both Java and Visual Basic. How many students know exactly one of the languages, Java and Visual Basic?

Question 9: (20 points)

(a) How many edges does $\overline{C_n}$ have? Your answer should be in terms of n .

(b) Draw $\overline{C_5}$.

(c) Prove that the only cyclic graph which may be isomorphic to its complement is C_5 .

(d) Does there exist a graph with a degree sequence of: 8, 6, 4, 2, 2, 2? Explain.

(e) Is C_i a subgraph of C_j , where $i < j$ and both i and j are integers and $i \geq 5$? Explain.
Note: Do not consider particular values of i and j .

Question 10: (20 points)

(a) How many edges does K_n have? And how many edges does $K_{m,n}$ have? Your answer should be in terms of n in the first and in terms of n and m in the second.

(b) What is the degree sequence of W_n ? Remember W_n is the Wheel Graph on n vertices. Your answer should be in terms of n .

(c) Decide whether the following graph is bipartite or not. Explain your answer. Is this graph isomorphic to the Utility Graph?

(d) Decide whether the following two graphs are isomorphic or not. Explain your answer.