CSCE 235 **Fall 2000**

Homework #2 (Due Monday, Sep 25, 00)

Question 1: Consider the poset $(A, \subseteq)$, where $A = \{\{1\}, \{1,2\}, \{1,3\}, \{1,2,4\}\}$.

(a) Find all maximum, minimum, maximal, and minimal elements of A.

(b) Is $(A, \subseteq)$ a totally ordered set? Explain

(c) Draw the Hasse Diagram of A.

(d) Find the *lub* and the *glb* of $\{1,2\}$ and $\{1,3\}$.

Question 2: Prove that:

$$(\sim (\sim (\sim a \vee b) \wedge a)) \equiv (a \Longrightarrow b).$$

Question 3: Do **only** one of the following three parts.

(a) Prove that the function $f : (\mathbb{N} \cup \{0\}) \times (\mathbb{N} \cup \{0\}) \longrightarrow \mathbb{N} \cup \{0\}$, defined by $f(k,n) = 2^k(2n+1) - 1$ is one-to-one.

(b) Prove that the function $f : (\mathbb{N} \cup \{0\}) \times (\mathbb{N} \cup \{0\}) \longrightarrow \mathbb{N} \cup \{0\}$, defined by $f(k,n) = 2^k(2n+1) - 1$ is onto. (*You have to be very careful here.*)

Notice that parts (a) and (b) prove that $\mathbb{N}$ has the same cardinality as $\mathbb{N} \times \mathbb{N}$, because the function f defined above is bijective (i.e. one-to-one and onto.)

(c) Use (a) and (b) to prove that if A and B are countably infinite sets, then so is $A \times B$.

Question 4: Do **only** one of the following three parts.

(a) Prove that if A and B are both countably infinite disjoint sets, then so is $A \cup B$. *Hint: Consider the function* $f : \mathbb{N} \longrightarrow A \cup B$*, which maps even natural numbers to* A

and odd natural numbers to B. All you need to do is to give a mathematical formula for f).

(b) Prove that the intervals (0,3) and $(3, \infty)$ have the same cardinality. *Hint: First show that (0,3) and (0,1) have the same cardinality by finding a one-to-one and onto function between them. Then show that $(3, \infty)$ has the same cardinality as $\mathbb{R}$ by finding a one-to-one and onto function between them. Finally, depend on the fact that (0,1) and $\mathbb{R}$ have the same cardinality. When you come up with the two functions mentioned above, you'll have the following:*

(0,3) has the same cardinality as (0,1), (0,1) has the same cardinality as $\mathbb{R}$, $\mathbb{R}$ has the same cardinality as $(3, \infty)$.

(c) Prove that the set of irrational numbers (i.e. $\mathbb{R} \setminus \mathbb{Q}$) is uncountable.

Notice that (c) implies that the cardinality of the set of irrational numbers is greater than the cardinality of the set of rational numbers.

Hints for some parts of questions 3 and 4: Remember

1. If $f : A \longrightarrow B$ is a bijective function, then $f^{-1} : B \longrightarrow A$ is defined and it is also bijective.

2. If $f : A \longrightarrow B$ and $g : B \longrightarrow C$ are bijective functions, then $g \circ f : A \longrightarrow C$ is also bijective.

Please if you need help stop by.

============== End of Homework 2 ==============

Questions to think about:

(1) Find a bijective function from $\mathbb{N} \times \mathbb{N}$ onto $\mathbb{Z}$.

(2) Find a bijective function from $(-1, 1)$ onto $(\frac{9999}{777}, \infty)$.

(3) Give a mathematical formula for a bijective function from $\mathbb{Z}$ onto $\mathbb{N}$.

(4) Find a bijective function from $\mathbb{N} \setminus \{1, 2, 3\}$ onto $(2\mathbb{N} - 1) \cup \{a, b, c\}$, where a, b, and c, are the letters a, b, c (This means $\mathbb{N} \cap \{a, b, c\} = \phi$.)