CSCE 235 **Exam II** **Nov 3, 2000**

Name : **SS#**

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Instructions: Do all of the following questions. Show your work and explain your answers.

Question 1: (15 points) Solve the following conqruence system:

$3x + 2y \equiv 2 \pmod 5$

$x + 2y \equiv 3 \pmod 5$

Question 2: (15 points) Solve the following equation:

$$x \equiv 2x(3x + 2) + 3 \pmod 7$$

Question 3: (20 points) Let a and b be nonzero integers and let $gcd(a, b) = g$. Also, let m and n be integers satisfying:

$$g = ma + nb.$$

(a) Let M_1 and N_1 be integers satisfying:

$$gcd(a, -b) = M_1 a + N_1(-b).$$

Express M_1 in terms of m and N_1 in terms of n.

(b) Express the least common multiple of $-a$ and $-b$ in terms of g, a, and b.

Question 4: (20 points) Solve the following recurrence relation by the method of *generating functions*:

$$a_n = 4a_{n-1},\ n \geq 1,\ a_0 = 3.$$

Question 5: (20 points) Solve the following recurrence relation by the method of *characteristic functions*:

$$a_n = 4a_{n-2} + 3^n,\ n \geq 2,\ a_0 = 1,\ a_1 = 2.$$

Question 6: (5 points)

Let N be an even natural number. Find an integer r, $0 \leq r < 4$, such that

$$1 + 2^{5N} \equiv r \pmod 4$$

Question 7: (5 points) Find the coefficient of x^n in the expansion of $\frac{3}{1-4x^2}$.